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F.Y B.Sc. (Computer Science) EXAMINATION, 2018

MATHEMATICS

Paper II

(MTC-102 : Algebra and Calculus)

(2013 PATTERN)

Time : Three Hours **Maximum Marks : 80**

N.B. :— (*i*) *All* questions are compulsory.

(*ii*) Figures to the right indicate full marks.

(*iii*) Neat diagrams must be drawn wherever necessary.

(*iv*) Use of non-programmable calculator is allowed.

1. Attempt any *eight* out of ten : [8×2=16]

(1) Find matrix and digraph for the following relation R on the set A = {1, 2, 3} :

R = {(1, 2), (1, 3), (3, 1), (3, 3)}.

(2) Define Euler's ϕ function. Find $\phi(250)$.

(3) Find values of the following expressions in $(\mathbf{Z}_5, +_5)$:

(*i*) $\overline{2}^3 + \overline{4} \times \overline{3}$

(*ii*) $\overline{2} \times \overline{4} - (\overline{3})^{-1}$.

P.T.O.

(4) State true or false with justification :

'The binary operation defined by $a * b = |a - b|$ on $\mathbf{Z}^+ \cup \{0\}$ is associative.'

(5) Let $f : R \to R$ be defined by $f(x) = 2x - 5$ and $g : R \to R$ be defined by $g(x) = \sin(x^2)$. Find $f \circ g(x)$, $g \circ f(x)$.

(6) Define elementary matrix. Find an elementary matrix E such that EA = I, where $A = \begin{bmatrix} 1 & 0 \\ -5 & 1 \end{bmatrix}$.

(7) Is the following matrix in reduced row echelon form ? Justify :

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

(8) State Maclaurin's theorem with Lagrange's form of remainder.

(9) If $y = \frac{1}{x^2 - x - 2}$, find y_n.

(10) Give geometrical interpretation of Rolle's mean value theorem.

2. Attempt any *four* out of six : [4×4=16]

(1) Find the remainder of 9^{153} when divided by 11.

(2) If $c | ab$ and $\gcd(b, c) = 1$, then prove that $c | a$.

(3) Prove by induction that $(x^n - y^n)$ is divisible by $(x - y)$ for $n \geq 1$.

(4) Express the following permutation on S_9 as a product of disjoint cycles :

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 1 & 4 & 3 & 6 & 7 & 5 & 9 & 8 \end{pmatrix}$$

Also find order of σ.

(5) Let G be the set of all non-zero real numbers and let $a * b = \frac{ab}{2}$. Show that (G, *) is an abelian group.

(6) Let A = {1, 2, 3} and R = {(1, 2), (2, 2), (2, 3), (3, 1), (3, 2)}. Find transitive closure of R using Warshall's algorithm.

3. Attempt any *two* out of three : [2×8=16]

(1) Define greatest common divisor (g.c.d.) of two integers. Find g.c.d. of 7260 and 1638. Also express it in the form $(7260)m + (1638)n$.

(2) (*i*) Prove that every subgroup of a cyclic group is cyclic.

(*ii*) Write all subgroups of $\mathbf{Z}_{10}$. Also write their generators.

(3) Define equivalence relation and equivalence class. Define a relation R on set of integers as follows :

'xRy if and only if $x + y$ is even'. Prove that R is an equivalence relation. Find distinct equivalence classes for this relation.

4. Attempt any *four* out of six : [4×4=16]

(1) Solve the following system by Gaussian elimination method :

$$x_1 + 2x_2 - 4x_3 + 3x_4 = 0$$
$$x_1 + 2x_2 - 2x_3 + 2x_4 = 0$$
$$2x_1 + 4x_2 - 2x_3 + 3x_4 = 0.$$

(2) Assuming the validity, obtain the Maclaurin series for $e^{\sin x}$.

(3) Evaluate :

$$\lim_{x \to 0} \sin x^{\tan x}.$$

(4) Let $f : [a, b] \to \mathbf{R}$ be continuous on $[a, b]$ and derivable on (a, b) such that $f'(x) = 0$ on (a, b), then prove that f is constant on $[a, b]$.

(5) Let

$$f(x) = -2 \sin x, \quad x \le -\frac{\pi}{2}$$
$$= \alpha \sin x + \beta, \quad -\frac{\pi}{2} < x < \frac{\pi}{2}$$
$$= \cos x, \quad x \ge \frac{\pi}{2}$$

Find the value of α and β if f is continuous everywhere.

(6) Let $f(x) = |x|$, $x \in \mathbf{R}$. Show that f is not differentiable at $x = 0$.

5. Attempt any *two* out of three : [2×8=16]

(1) Using LU decomposition, solve the following system :

$$\begin{bmatrix} 3 & -6 & -3 \\ 2 & 0 & 6 \\ -4 & 7 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -3 \\ -22 \\ 3 \end{bmatrix}.$$

(2) State Leibnitz's theorem. If $y = a \cos(\log x) + b \sin(\log x)$, then show that $x^2 y_2 + x y_1 + y = 0$. Hence prove that :

$$x^2 y_{n+2} + (2n + 1) x y_{n+1} + (n^2 + 1) y_n = 0.$$

(3) State and prove Cauchy's mean value theorem. Verify it for the functions $f(x) = x^2$ and $g(x) = x^4$ in $[a, b]$ where a and b are positive real numbers.